

- ABHISEKH SANKARAN, BHARAT ADSUL, AND SUPRATIK CHAKRABORTY,
A generalization of the Łoś-Tarski preservation theorem.
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We present new parameterized preservation properties that provide, for a natural number k as a parameter, semantic characterizations of the $\exists^k\forall^*$ and $\forall^k\exists^*$ prefix classes of FO sentences, over the class of all structures and for arbitrary finite vocabularies. Unlike preservation properties in the literature [1] that characterize the $\exists^*\forall^*$ and $\forall^*\exists^*$ prefix classes as a whole, our properties enable finer characterizations by relating the count of quantifiers in the leading block of $\exists^*\forall^*$ and $\forall^*\exists^*$ sentences, to natural model-theoretic properties. As a consequence, we get a parameterized generalization of the classical Łoś-Tarski preservation theorem for sentences, in both forms of the latter, substructural and extensional. We call our characterizations collectively as the *generalized Łoś-Tarski theorem for sentences*. We also generalize the extensional form of the generalized Łoś-Tarski theorem to theories over arbitrary finite vocabularies, by giving a semantic characterization of theories of $\forall^k\exists^*$ sentences.

Unlike any of the aforementioned preservation properties in the literature, our preservation properties are combinatorial and finitary in nature, and remain non-trivial over finite structures as well. There has been recent interest in the context of finite model theory, in studying preservation theorems over various “well-behaved” classes of finite structures. In particular, Atserias, Dawar and Grohe showed in [2] that under suitable closure assumptions, classes of structures that are acyclic, of bounded degree or of bounded tree-width admit the Łoś-Tarski theorem for sentences. We identify many interesting classes of finite structures that admit the generalized Łoś-Tarski theorem for sentences, and furthermore, admit it in effective form. Specific examples include the classes of words, trees (as partial orders), structures of bounded tree-depth, grids of bounded dimension, various subclasses of co-graphs, etc. Some of these, like structures of bounded tree-depth, were earlier not known to even satisfy the Łoś-Tarski theorem.

In summary, our preservation properties yield new preservation theorems in the contexts of both, classical model theory and finite model theory.

[1] C. C. CHANG AND H. J. KEISLER, *Model Theory*, Elsevier Science Publishers, 3rd ed., 1990.

[2] A. ATSERIAS, A. DAWAR AND M. GROHE, *Preservation under Extensions on Well-Behaved Finite Structures*, *SIAM Journal of Computing*, vol. 38 (2008), no. 4, pp. 1364–1381.